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Short communication

A critical review of using the Peukert equation for determining the remaining capacity of lead-acid and lithium-ion batteries

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Abstract

In many applications it is essential to predict the remaining capacity of a battery reliably, accurately and simply. Several existing techniques for predicting the remaining capacity of a lead-acid battery discharged with a variable current are based on variants of Peukert's empirical equation, which relates the available capacity to a constant discharge current. This paper presents a critical review of these techniques in the light of experimental tests that were carried out on two lead-acid commercial batteries. The relevance of these Peukert's equation based techniques to lithium-ion batteries is also discussed in the light of tests carried on a lithium-ion power battery. The basic conclusion of the paper is that Peukert's equation cannot be used to predict the state of charge of a battery accurately unless it is discharged at a constant current and constant temperature.

Keywords: Battery testing; State-of-charge; Modelling; Lead-acid battery; Lithium-ion battery

1. Introduction

Prediction of state-of-charge (SOC) of a battery is vital in many applications. Many scientists and engineers [1–8] base their methods for SOC prediction on Peukert's equation [9], which relates the available capacity of a lead-acid battery to discharge rate, for a constant current discharge. As the discharge current in most applications is variable, several methods were proposed [1–8,11] to adapt Peukert's equation to a variable current discharge. However, upon closer examination, these techniques can be shown (see later sections) to produce different and sometimes confusing results regarding the state of charge of a battery.

This paper presents a review of Peukert's findings based on the original paper that was published in 1897 [9] in the light of tests carried out on two commercial lead-acid batteries. The alternative techniques proposed in the literature for using variants of Peukert's empirical equation to predict the state of charge of a lead-acid battery discharged with a variable current are then critically reviewed. The paper also discusses the relevance of Peukert's findings and those techniques to lithium-ion batteries in the light of tests on a commercial lithium-ion battery.

2. A review of Peukert's findings

Peukert performed constant current discharge tests on several different lead-acid batteries from different manufacturers. He found that a simple equation was sufficient to put capacity and discharge rate into relation for all lead-acid batteries [9]:

$$I^{\mathrm{pc}}t = \text{constant} \tag{1}$$

where I is the discharge current, t the maximum discharge time and pc is the "Peukert coefficient" (usually between 1 and 2) unique to a battery of a certain make and model.

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A Peukert-coefficient of pc = 1, for example, means that the accessible total capacity of that battery does not depend on the discharge rate, which is not true for real lead-acid batteries which usually have a pc > 1. This simple equation enables the calculation of the available discharge time for a given battery with a certain Peukert coefficient discharged with a constant current load.

Most battery manufacturers specify the capacity of their batteries for a certain discharge time of n (h), for example, $C_n = 100$ Ah [10]. This means that the battery will deliver 100 Ah if discharged at such a rate that the discharge time is n hours. Using this example, if $n = 20$ (h), the rate would be $I_{20} = 5$ A. The Peukert equation can be used for calculating the available capacity C_{n1} at a different discharge rate I_{n1} using the following equation which is derived in Appendix A:

$$C_{n1} = C_n \left(\frac{I_n}{I_{n1}} \right)^{\mathrm{pc}-1} \quad (2)$$

The total discharge time will be $n1$ h. Peukert found that pc was about 1.47 on average for available lead-acid batteries at that time. Modern batteries have better coefficients. This means that the available capacity at a constant discharge current becomes less if the discharge rate increases. The loss of capacity at a high discharge rate was explained by Peukert to be due to "a poorer utilisation of the electrode surface". However, based on latest research [14–16], the loss of capacity at a high discharge rate can be explained to be due to a decrease of the number of active centres in the positive active material and a rapid increase in the resistance of the interface between the grid and the lead dioxide active material.

Peukert's equation should be interpreted with care. It should not be understood to mean that when a battery is discharged fully at a certain high current discharge rate that it is completely empty. In fact it is well-known that a seemingly empty battery discharged at a high current will still have some available capacity at a lower discharge current [10], [12]. In fact [10] suggests that if a battery was discharged at successively decreasing rates, the total Ah capacity obtained from a battery will be the same as that obtained from using a constant low current discharge. However, this paper presents results that suggest that there is a net loss of available capacity when a battery is discharged at a high rate, followed by successive low discharge rates compared to a battery discharged at a low current rate from the start.

3. Experiments

Battery discharge tests at specified rates as described in the following sections are carried out on three specimens which are an aged 12 V 65 Ah sealed lead-acid battery (battery BLA1), a new sealed lead-acid with 17.2 Ah (battery BLA2) and an aged 50 Ah lithium-ion cell (battery Blion). All tests are carried out at room temperature using a commercial battery tester. Before a test the cells are fully charged as recommended by the manufacturers.

3.1. Lead-acid battery tests

In the first set of tests, a typical test consisted of two test sections. In the first section, the test specimen is fully discharged at a high rate I_{high}. After a waiting time, t_{wait} it is further discharged with a small current I_{low}. After fully recharging, in the second test section, the same cell is discharged with the low rate I_{low} from the beginning. The waiting time t_{wait} after the first high-rate discharging is 6 h, so that the total discharge time in the second test section is similar to the discharge time in the first test section. The aim is to minimise the impact of mass-transportation limitations [11] and to give time for the hydration of active centres in the positive electrode—see, [16]. This whole test is repeated four times with identical charging algorithms and identical waiting times between charging and discharging and between the tests and between the test sections. The charged and discharged Ahs are counted separately throughout all tests.

Fig. 1 shows the battery voltage versus depth of discharge (DOD) for the BLA1 lead-acid battery. Two out of four tests are shown for clarity; the other tests produce almost identical results. The two upper-right curves represent two cycles of the second test section: the discharges at a small current of 5 A from the beginning. The discharge ends at point C with 67.6 Ah discharged in total. The voltages relax to 11.58 V in point F after 1 h. Both curves are almost identical. This demonstrates the good repeatability of the test.

The two lower curves represent two cycles of the first test section: the discharges start with a high current of 50 A until the cut-off voltage of 10.0 V is reached at point A. A discharge of 44.2 Ah occurs up to this point. Within the resting period of 6 h, the voltage relaxes to 12.13 V (point D). Following this stage is the next continuous discharge at a lower rate of 5 A until the battery reaches the cut-off voltage again at point B, having now discharged a total of 64.3 Ah. The voltages relax up to 11.71 V at point E in the subsequent pause of 1 h. Again, both curves are almost identical, demonstrating good repeatability.

The BLA2 sealed lead-acid battery is tested in the same way in order to find out whether the behaviour is unique to a certain battery design or whether it may be general to the lead-acid chemistry. Fig. 2 shows the test results with the BLA2 sealed lead-acid battery. The test undertaken is generally the same as the tests with the BLA1 battery. However, the battery is discharged to different cut-off voltages, depending on the discharge current and following the recommendations of the manufacturer. Additionally, four different discharge rates are tested instead of only two, and the highest discharge rate is 40 times higher than the lowest, instead of being only 10 times higher.

Figs. 1 and 2 confirm the well-known fact [10] that a battery fully discharged at a high current rate can be discharged further at a lower current rate. However, they also reveal that

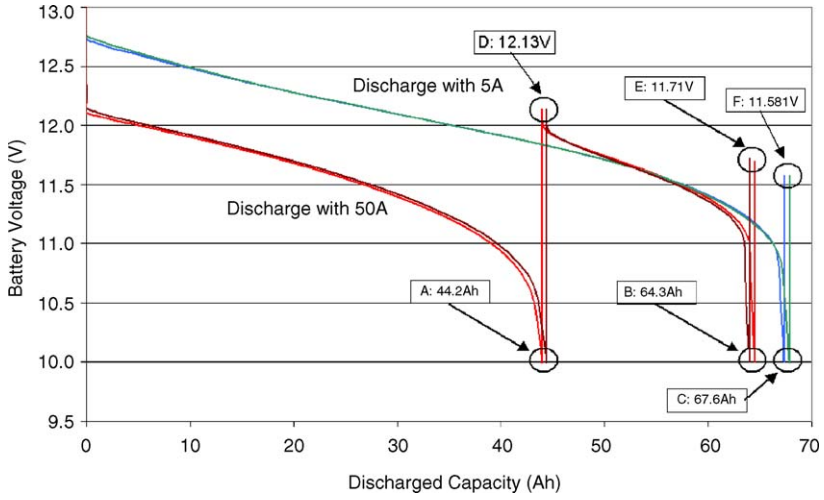

Fig. 1. First set of discharge tests on a 65 Ah sealed lead-acid battery BLA1 (black and white).

there is a net loss of Ah capacity that is caused by discharging the battery at a high rate first before the low current discharge.

A second set of tests was also carried out on the batteries as shown in Fig. 3. The test procedure is very similar to the previous test. The only difference is that another discharging at the same high rate is performed after the waiting time of 6 h and before the subsequent discharge at the lower rate.

The tests in Fig. 3 present evident that a battery can be discharged further at the same rate when it is left to rest. This may be explained according to [14,15], to be due to the reformation of the hydrated gel zones in the electrode active centres during the waiting period. However, there is still a net loss of capacity compared to the low current discharge rate case, which is similar to that in Figs. 1 and 2. This net loss of capacity may be explained according [14–16] to be due to stoichiometric changes in the interface between the grid and the lead dioxide active mass at high current discharge which leads to an increase in the resistance of the interface and hence a net loss of capacity when the battery is further discharged at the low rate.

3.2. Lithium-ion battery tests

Equivalent tests to those described in Figs. 1 and 2 were carried out on a large high-energy lithium-ion cell (Blion) with 50 Ah capacity. Fig. 4 shows the results with an aged 50 Ah high-energy lithium-ion cell. The top graphs show the cell voltage versus depth of discharge. The cell was discharged with a high rate of 50 A till the cut-off voltage of 2.8 V is reached at point B. After a waiting time of 6 h, it was further discharged with a small current of 5 A till the cut-off voltage is reached at point C. This is compared with a discharge at the lower current of 5 A from the beginning till the cut-off voltage is reached in point A. The bottom graphs in Fig. 4 show the cell temperatures during the same tests. The procedures are repeated three times each time yielding very similar curves; however, in order to achieve better readability, not all of the test results are presented in Fig. 4.

Fig. 4 reveals that for the tested large high-energy lithium-ion cell, the dischargeable capacities are between 30 and 32 Ah in all tests regardless the discharge rate. This can be

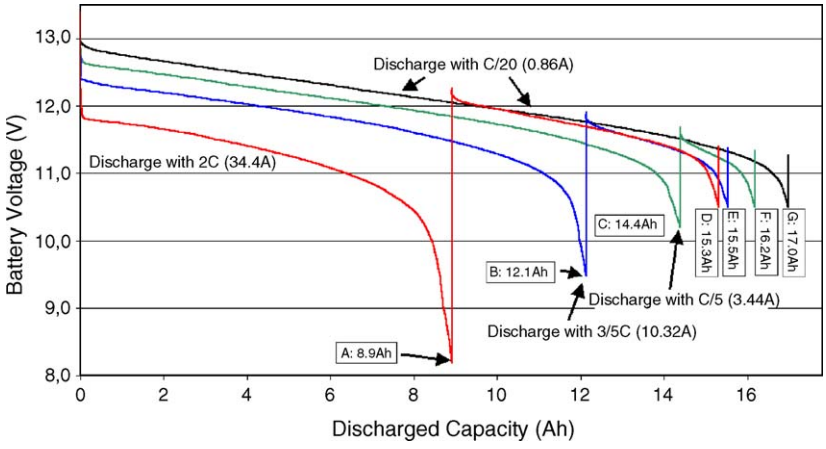

Fig. 2. Discharge tests on a 17 Ah sealed lead-acid battery BLA2 (black and white).

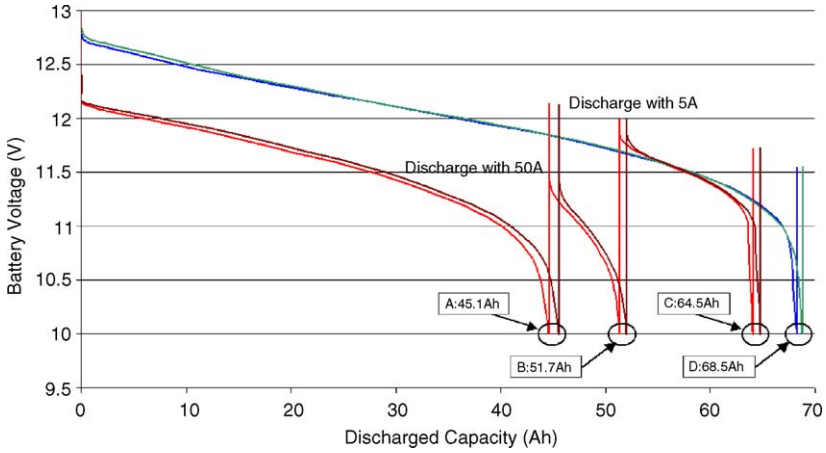

Fig. 3. Second set of discharge tests on a 65 Ah sealed lead-acid battery BLA1 (black and white).

explained with the rise in cell temperature to almost 55 °C during the continuous high rate discharge, which is known to enhance the performance of a lithium-ion cell. In contrast, the temperature at low rate discharge stays at about 25 °C as shown in Fig. 4. If the battery is discharged at a high current, while the battery temperature is maintained at 25 °C, then it is expected that the available capacity will be reduced. Clearly, the battery temperature rise, which is a function of

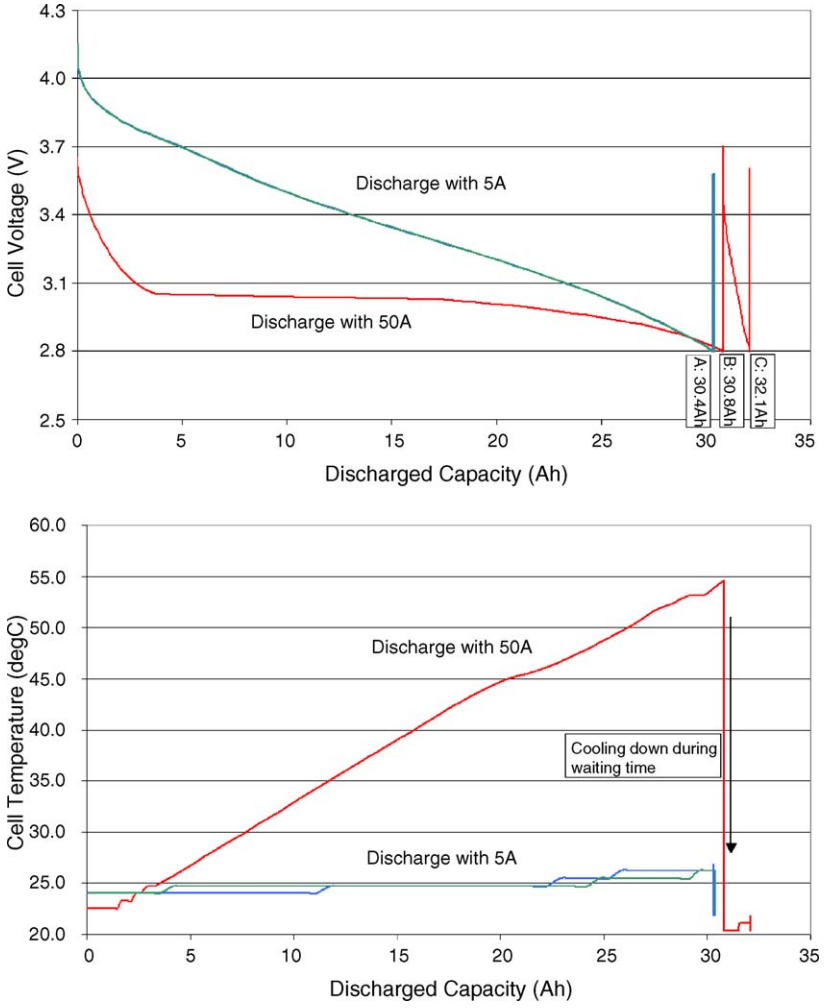

Fig. 4. Cell voltage and temperature during the "Passivation test" on a lithium-ion cell Blion (black and white).

environmental conditions and discharge duty cycle and the design of the battery, is an important factor that should be taken into account when making estimates of the remaining capacity in this type of battery.

4. Review of techniques using Peukert's equation to calculate remaining capacity

As mentioned earlier, many researchers developed techniques adapting Peukert's equation to estimate the remaining capacity in a battery when discharged using a variable current. For example [2–5,7], apply the Peukert equation to varying currents by basically calculating an "effective discharge current" $I_{\text{effective}}$ based on Peukert's equation:

$$I_{\text{effective}} = I\left(\frac{I}{I_{\text{nominal}}}\right)^{\text{pc}-1} \tag{3}$$

In this equation, I is the actual current and I_{nominal} is the nominal current for which the nominal capacity is given by the manufacturer. The "effectively discharged capacity" is then calculated by numerical integration of this effective current over time:

$$C_{\text{dch,effective}} = \sum (I_{\text{effective}}\,\Delta t) \tag{4}$$

The remaining capacity is then determined by subtracting this effectively discharged capacity from the nominal capacity:

$$C_{\text{remaining}} = C_{\text{nominal}} - C_{\text{dch,effective}} \tag{5}$$

These methods would determine zero remaining capacity after a "full" discharge at a high current (point A in Fig. 1). However, our tests reveal that the battery can still deliver a small amount of capacity at the same current after a waiting time. A substantial capacity may be discharged at a lower current.

Other methods [1,6] set I in Eq. (3) to be the average discharge current (usually during the last 5 min) to calculate an effective current, which multiplied by the total discharge time to calculate the discharged capacity so far. Eq. (5) is then used to calculate the remaining capacity. However, the "fuel-gauge" based on this method would show inconsistent results: when no current is drawn the remaining capacity would go slowly up with time, and it would go down steeply when drawing high currents, which might confuse the user. Like the effective current method, the average current technique tends to underestimate the remaining capacity in a battery.

For lithium-ion batteries, the remaining capacity estimation method should significantly take into account the temperature of the battery, which is a function of the load duty cycle, battery design and environmental conditions. The Peukert equation is not applicable.

5. Conclusions

Test results were presented which show that a seemingly fully discharged (at a high rate) lead-acid battery may be discharged further after a period of rest, which allows the active centres in the electrodes to recover (hydrate). However, if a battery is discharged at a high rate followed by discharge at a lower rate, the capacity obtained will be less than that obtained from the battery if it was discharged from the start at a low rate. This is thought to be due to chemical and structural changes in the active material grid interface that occur during a high current discharge, which increases the resistance of the interface, thus leading to a net loss of capacity.

The capacity obtained from a lithium-ion battery is strongly dependent on temperature, which is in turns dependent on the rate of discharge. At a high current discharge rate, the temperature of the battery may increase considerably, thus increasing the available capacity.

Peukert's equation is strictly applicable to batteries discharged at constant temperature and constant discharge current. When applied to a battery with a variable discharge rate and changing operating temperature using average or effective current, it generally results in an underestimation of the remaining capacity.

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Appendix A. Derivation of Eq. (2)

From (1)

$$I_n^{\text{pc}} t_n = I_{n1}^{\text{pc}} t_{n1} \tag{A1}$$

The above equation can be written as

$$I_n t_n I_n^{\text{pc}-1} = I_{n1} t_{n1} I_{n1}^{\text{pc}-1} \tag{A2}$$

Substituting $I_n t_n = C_n$ and $I_{n1} t_{n1} = C_{n1}$ yields

$$C_n I_n^{\text{pc}-1} = C_{n1} I_{n1}^{\text{pc}-1} \tag{A3}$$

which can be arranged to yield Eq. (2).

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